$$AB = \frac{CD}{(1 - v^2/c^2)^{1/2}} \quad \Rightarrow \quad \Delta x = \frac{\Delta x'}{(1 - v^2/c^2)^{1/2}}$$

Hence, we have the length contraction formula. It should be noted that according to S, events A and B are not simultaneous and comparison of length is only possible because of the arbitrary method of linking events by positional correlation.

Phase Difference in the Synchronization of Clocks

We are now going to demonstrate quantitatively the reason for the phase difference term:

$$\frac{-\ vx/c^2}{(1 - v^2/c^2)^{1/2}} \quad \text{in the equation:} \quad t' = \frac{t - vx/c^2}{(1 - v^2/c^2)^{1/2}}$$

The presence of the term means that clocks further away in the positive x direction on the moving reference frame S′ will show earlier times.

Fig. 3a shows the paths of light rays originating at event O and travelling to events A and B as seen by stationary reference frame S. Fig. 3b shows the paths of the same light rays according to S′. The distances traversed by the light rays from O to A and from O to B are the same (= ½) according to S, and so A and B are considered as simultaneous events. Also, according to S, the reference frame S′ has moved a distance ½v/c during the time the light ray took to travel from O to A (or from O to B). The observer on S′ would say therefore that the distances traversed by the two light rays are different. For him the distances would be O′A and O′B where O′ is the event considered by S′ to be positionally correlated to event O, and considered by S to be simultaneous to A and B.

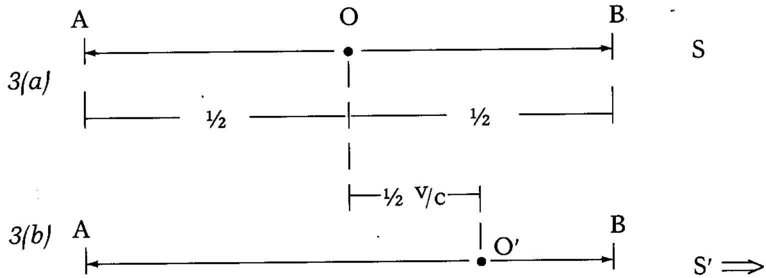

Fig. 3. Fig. 3(a) shows the paths of light rays from event O to events A and B according to inertial reference frame S. Fig. 3(b) shows the paths of the same two light rays according to another inertial reference frame S′ moving with velocity v relative to S. O′ is the event considered by S′ to be positionally correlated to event O, and considered by S to be simultaneous to A and B. The values of the distances shown in the diagram are those according to S. The equivalent distances according to S′ would be greater by a factor of $(1 - v^2/c^2)^{-1/2}$. As the distance traversed by the light rays to A and to B are the same according to S, the events A and B are considered simultaneous. However, according to S′, the distances traversed by the light rays are not the same, so A and B are not considered simultaneous. Hence there is a phase difference in the synchronization of clocks.

From man's arbitrary definition of time, the distance the light ray travels in his demarcated space is directly the measure of the elapsed time according to him (however, the unit used for distance is smaller than the unit for time by the factor of c). Therefore, according to S′, the time between event O and event B would be:

$$\frac{\tfrac{1}{2} - \tfrac{1}{2}v/c}{c(1 - v^2/c^2)^{1/2}}$$

The Lorentz factor: $(1 - v^2/c^2)^{-1/2}$ is necessary because, as shown earlier, the distance according to S′ is greater by that proportion in comparison to that according to S. The time between event O and event A according to S′ would be:

$$\frac{\tfrac{1}{2} + \tfrac{1}{2}v/c}{c(1 - v^2/c^2)^{1/2}}$$

The time difference according to S′ between event B and event A would then be:

$$\frac{\tfrac{1}{2} - \tfrac{1}{2}v/c}{c(1 - v^2/c^2)^{1/2}} - \frac{\tfrac{1}{2} + \tfrac{1}{2}v/c}{c(1 - v^2/c^2)^{1/2}} = \frac{-\,v/c^2}{(1 - v^2/c^2)^{1/2}}$$

This difference is observed over a distance of 1 according to S. Therefore, the time difference in the clock readings over the distance x becomes:

$$\frac{-\,vx/c^2}{(1 - v^2/c^2)^{1/2}}$$

Hence we have the phase difference term.

From all the arguments given above, we can now readily construct the Lorentz transformation equations. Hence we have shown that they are a consequence of man's arbitrary definitions of time and space as outlined in this paper.