caused by a shift to a reference frame having the same velocity as the particle, and then add on the changes brought about by a subsequent shift to the actual final reference frame you want to consider. Other appropriate combinations may of course be employed.

For conceptual purposes, we shall now consider the case where we keep the time and space coordinates unchanged, when we shift from one inertial reference frame to another. We want to determine the change in the time and space coordinates according to the initial reference frame $S$, which would be required to correlate with the new event designated by the new reference frame $S'$. ($S'$ moves with velocity $v$ relative to $S$ along the $x$-axis.)

We therefore set:

$$t' = t_0, \quad x' = x_0, \quad y' = y_0, \quad z' = z_0.$$  

From the Lorentz transformation equations, we now obtain:

$$t = t_0 + \frac{v^2 x_0}{c^2} \sqrt{1 - \frac{v^2}{c^2}},$$  

$$x = \frac{x_0}{1 - \frac{v^2}{c^2}}, \quad y = y_0, \quad z = z_0.$$  

With all the above results, we are now ready to finally "solve" the famous twin paradox.

The twin paradox poses a problem in interpretation mainly because of man's assumption that time is a real entity that is pervasive. This notion then compels him to connect all the events experienced by each of the twins into simultaneous sets. He is then alarmed to find that, according to the "travelling" twin, there are events experienced by the "stationary" twin which cannot be reasonably linked by simultaneity, and may even seem to have disappeared (i.e. there has been a sudden time leap). If we accept that time and space are not real pervasive entities but only arbitrary abstractions, the problem ceases to exist, as we can now demonstrate.

To simplify matters, we shall consider the case where no acceleration is involved and, instead of using twins, we shall compare the times shown on clocks. Consider Fig. 4. Rocket A approaches a stationary position $E$ at constant velocity $v$. As
where \( \Delta t \) is the time lapse according to \( E \), between rocket \( A \) passing \( E \) and rocket \( B \) arriving at \( E \). \( \Delta t' \) is the corresponding time lapse recorded by the clocks on the rockets.

Now, we shall look at the situation according to the observers on rocket \( A \) and then on rocket \( B \). We shall divide our analysis into three phases as follows:

Phase 1: between \( A \) passing \( E \) and \( A \) meeting \( B \).
Phase 2: the shift of reference frame from \( A \) to \( B \).
Phase 3: between \( B \) passing \( A \) and \( B \) meeting \( E \).

We denote \( \Delta t_1 \), \( \Delta t_2 \), \( \Delta t_3 \) as the time lapses at \( E \) and \( \Delta t_1' \), \( \Delta t_2' \), \( \Delta t_3' \) as the time lapses on the rockets corresponding to phases 1, 2, and 3 respectively.

In phase 1, the observer on \( A \) will have considered the clock on \( E \) to have run slower by a factor of \( (1 - v^2/c^2)^{1/2} \). Due to the length contraction effect of the Lorentz transformation equations, he will also have considered the distance travelled in phase 1 to be \( x(1 - v^2/c^2)^{1/2} \). Hence, for phase 1, we obtain:

\[
\Delta t_1 = \frac{x}{v}(1 - \frac{v^2}{c^2})
\]

\[
\Delta t_1' = \frac{x}{v}(1 - \frac{v^2}{c^2})^{1/2}
\]

In phase 2, we have to consider the effect on the time and space coordinates, brought about by the abrupt shift in reference frame from rocket \( A \) to rocket \( B \). We shall evaluate the effects in two stages.

In the first stage, we determine the effects resulting from a shift from \( A \) to the reference frame stationary relative to \( E \). We therefore apply the results from case 2(b) of section IV and obtain:

\[
\Delta t_1 = \frac{vx}{c^2}
\]
where $\Delta t$ is the time lapse according to $E$, between rocket $A$ passing $E$ and rocket $B$ arriving at $E$. $\Delta t'$ is the corresponding time lapse recorded by the clocks on the rockets.

Now, we shall look at the situation according to the observers on rocket $A$ and then on rocket $B$. We shall divide our analysis into three phases as follows:

Phase 1: between $A$ passing $E$ and $A$ meeting $B$.
Phase 2: the shift of reference frame from $A$ to $B$.
Phase 3: between $B$ passing $A$ and $B$ meeting $E$.

We denote $\Delta t_1$, $\Delta t_2$, $\Delta t_3$ as the time lapses at $E$ and $\Delta t_1'$, $\Delta t_2'$, $\Delta t_3'$ as the time lapses on the rockets corresponding to phases 1, 2 and 3 respectively.

In phase 1, the observer on $A$ will have considered the clock on $E$ to have run slower by a factor of $(1 - \nu^2/c^2)^{\frac{1}{2}}$. Due to the length contraction effect of the Lorentz transformation equations, he will also have considered the distance travelled in phase 1 to be $x(1 - \nu^2/c^2)^{\frac{1}{2}}$. Hence, for phase 1, we obtain:

$$\Delta t_1 = \frac{x}{\nu}(1 - \nu^2/c^2)$$

$$\Delta t_1' = \frac{x}{\nu}(1 - \nu^2/c^2)^{\frac{1}{2}}$$

In phase 2, we have to consider the effect on the time and space coordinates, brought about by the abrupt shift in reference frame from rocket $A$ to rocket $B$. We shall evaluate the effects in two stages.

In the first stage, we determine the effects resulting from a shift from $A$ to the reference frame stationary relative to $E$. We therefore apply the results from case 2(b) of section IV and obtain:

$$\Delta t_1 = \frac{vx}{c^2}$$
\( L_1 = x \)

where \( \Delta \tau_1 \) is the proper time lapse (according to E) between the event on E considered simultaneous to A and that considered simultaneous to the new reference frame. \( L_1 \) is the distance to E according to the new reference frame.

For the second stage, we have to evaluate the effect on the time and space coordinates resulting from an abrupt shift from the reference frame stationary relative to E to the reference frame of rocket B. We now apply the results from case 1(b) of section IV and obtain:

\[ \Delta \tau_2 = \frac{vx}{c^2} \]

\[ L_2 = x[1 - \frac{v^2}{c^2}]^{1/2} \]

where \( \Delta \tau_2 \) is the proper time lapse (according to E) between the event on E considered simultaneous to the reference frame stationary relative to E and that considered simultaneous to B. \( L_2 \) is the distance to E according to rocket B. We note that this is the same as that according to rocket A.

Hence, for phase 2, we obtain:

\[ \Delta t_2 = \Delta \tau_1 + \Delta \tau_2 = 2vx/c^2 \]

\[ \Delta t_2' = 0 \]

The fact that \( \Delta t_2 \) is not zero means that there has been an apparent instantaneous jump in time at E the moment the switch of reference frames occurred. It should also be noted that \( \Delta t_2 \) is directly proportional to the distance between E and the position where the rockets passed each other. This is one of the effects which many either had not considered or had found difficult to accept previously. However, once we understand that time and space are only arbitrary concepts and that all these changes are merely the result of an alteration in the method of designating events, such effects become perfectly acceptable.

In phase 3, the conditions are similar to those in phase 1 and we have:

\[ \Delta t_3 = \frac{x}{v}[1 - \frac{v^2}{c^2}] \]

\[ \Delta t_3' = \frac{x}{v}[1 - \frac{v^2}{c^2}]^{1/2} \]

Thus, according to the observers on A and B, the total time lapse at E is:

\[ \Delta t_1 + \Delta t_2 + \Delta t_3 = 2x/v \]

and the total time lapse according to the clocks on the rockets is:

\[ \Delta t_1' + \Delta t_2' + \Delta t_3' = (2x/v)[1 - \frac{v^2}{c^2}]^{1/2} \]

These results are exactly the same as those obtained from the point of view of the observer on E. Hence there is no paradox as both sets of observers agree on the results (i.e., both agree that the time lapse on the rockets is less than that on E).
$\Delta t_2 = \frac{v x}{c^2}$

$L_2 = x (1 - \frac{v^2}{c^2})^{\frac{1}{2}}$

where $\Delta t_2$ is the proper time lapse (according to E) between the event on E considered simultaneous to A and that considered simultaneous to the reference frame of rocket B. $L_2$ is the distance to E according to the reference frame of rocket B. We now apply the results from case 1(b) of section IV and obtain:

For the second stage, we have to evaluate the effect on the time and space coordinates resulting from an abrupt shift from the reference frame stationary relative to E to the reference frame of rocket B. We now apply the results from case 1(b) of section IV and obtain:

$\Delta t_2 = \frac{v x}{c^2}$

$L_2 = x (1 - \frac{v^2}{c^2})^{\frac{1}{2}}$

where $\Delta t_2$ is the proper time lapse (according to E) between the event on E considered simultaneous to the reference frame stationary relative to E and that considered simultaneous to B. $L_2$ is the distance to E according to rocket B. We note that this is the same as that according to rocket A.

Hence, for phase 2, we obtain:

$\Delta t_2 = \Delta t_1 + \Delta t_2 = 2\frac{v x}{c^2}$

$\Delta t_2' = 0$

The fact that $\Delta t_2$ is not zero means that there has been an apparent instantaneous jump in time at E the moment the switch of reference frames occurred. It should also be noted that $\Delta t_2$ is directly proportional to the distance between E and the position where the rockets passed each other. This is one of the effects which many either had not considered or had found difficult to accept previously. However, once we understand that time and space are only arbitrary concepts and that all these changes are merely the result of an alteration in the method of designating events, such effects become perfectly acceptable.

In phase 3, the conditions are similar to those in phase 1 and we have:

$\Delta t_3 = \frac{x}{v} (1 - \frac{v^2}{c^2})$

$\Delta t_3' = \frac{x}{v} (1 - \frac{v^2}{c^2})^{\frac{1}{2}}$

Thus, according to the observers on A and B, the total time lapse at E is:

$\Delta t_1 + \Delta t_2 + \Delta t_3 = \frac{2x}{v}$

and the total time lapse according to the clocks on the rockets is:

$\Delta t_1' + \Delta t_2' + \Delta t_3' = \frac{2x}{v} (1 - \frac{v^2}{c^2})^{\frac{1}{2}}$

These results are exactly the same as those obtained from the point of view of the observer on E. Hence there is no paradox as both sets of observers agree on the results (i.e. both agree that the time lapse on the rockets is less than that on E).