IV

Acceleration and the Effects of Changing Inertial Reference Frames

It is clearly possible for an inertial observer to set up a uniform and pervasive system of time and space coordinates. It is not clear, however, whether a meaningful system of time and space could be devised for an accelerating frame of reference, as time and space are really arbitrary conceptions designed (though inadvertently) by man for basically inertial systems. The reference frame of the earth may be considered virtually as an inertial system as the effect of its acceleration and its gravitational field on the determination of time and space is relatively insignificant.

Any attempt to create a system of time and space for an accelerating reference frame would most likely require further arbitrary rules on how certain aspects should be defined, and this may render it of dubious scientific value. For instance, the acceleration of a system may be maintained by a force applied at one position. Due to the time required for the transmission of this force, the spatial configuration of solid state matter in this accelerating system would be different from that in an inertial system travelling at the same velocity at that instant. We are then forced to make an arbitrary choice as to which solid state matter to utilize in demarcating space.

It may be better, then, to designate the events in an accelerating system in terms of the time and space coordinates defined for an assigned inertial reference frame, rather than to attempt formulating its own arbitrary system of time and space. Also, instead of trying to determine "aging" (the time experienced by an object) which is really an abstraction, we may describe what happens to accelerating objects directly in terms of electromagnetic phenomena which is the underlying reality behind the concepts of time and space. In other words, as time and space are only arbitrary concepts devised by man, it is not scientifically essential for an accelerating system to have its own time and space, as previously thought.

However, as time and space can be clearly defined for all inertial systems, it is useful to determine how the time and space coordinates designating events would alter if we changed inertial reference frames. It will then be possible to cope with temporarily accelerating systems by just considering the changes arising from a shift from its initial inertial frame to its eventual inertial frame. In other words, we do not try to devise any time and space coordinate system for the observer while he is accelerating, but we can define clearly what has happened to his time and space coordinates of events, when he reaches his final inertial reference frame. To link up the initial and eventual reference frames we may use a third arbitrary inertial frame, and work out the changes in the time and space coordinates of events in two stages. The end result will not be affected.

In order to be able to determine the time and space coordinate changes of all inertial particles brought about by an abrupt shift in inertial reference frame, we have to consider two different cases. In both of these we will assign the direction of the line of motion between the initial and the eventual inertial reference frame as being on the x-axis, and we set the coordinates of the event on the particle simultaneous to the initial reference frame (at the time of the shift of inertial frame) as t_0, x_0, y_0, z_0.

Case 1

Here we will examine the change in the time and space coordinates of a theoretical point particle A which is not moving along the x-axis according to the initial reference frame S. The abrupt shift is to another inertial reference frame S′ moving with velocity v on the x-axis. There are two events on the particle A to consider: one simultaneous to S and the other simultaneous to S′ at the time of the reference frame change. These are separate events.

1(a)

First, we will evaluate the change in the time and space coordinates of the event at A which is considered simultaneous according to S. Let t, x, y, z and t′, x′, y′, z′ be the coordinates according to S and S′ respectively. We have therefore:

$$t = t_0, \quad x = x_0, \quad y = y_0, \quad z = z_0.$$

Once we shift to the reference frame S′, we have, from applying the Lorentz transformation equations:

$$t' = t_0 - \frac{(v/c^2)x_0}{(1 - v^2/c^2)^{1/2}},$$

$$x' = \frac{x_0}{(1 - v^2/c^2)^{1/2}}, \qquad y' = y_0, \qquad z' = z_0.$$

This means that the moment we shift to a new inertial reference frame, the event on the particle which we had considered simultaneous suddenly becomes an event of the past. A span of time has apparently disappeared! Furthermore, the event is suddenly being designated at a different point in space. These effects would have been difficult to comprehend previously. However, with the knowledge that time and space are really arbitrary conceptions of man, they now become perfectly acceptable. What has changed is merely man's arbitrary method of designating events. Nothing strange such as "time or space jumps" has actually happened to the particle.

1(b)

We now evaluate the time and space coordinates of the event at the particle considered simultaneous to the eventual inertial frame S′ at the time of the abrupt shift in reference frame. We obtain:

$$t = t_0 + (v/c^2)x_0, \qquad x = x_0,$$

$$t' = t_0, \qquad x' = x_0(1 - v^2/c^2)^{1/2},$$

and

$$y = y' = y_0 + u_y\Delta t, \qquad z = z' = z_0 + u_z\Delta t$$

where u_y and u_z are, respectively, the y and z velocity components of particle A according to S, and $\Delta t = (v/c^2)x_0$ is the change in the time coordinate according to S, between this event considered simultaneous to S′ and that — as in 1(a) — considered simultaneous to S.

An important value to note here is the change in the proper time on particle A (Δt_A) between these two events:

$$\Delta t_A = \Delta t = (v/c^2)x_0$$

This means that immediately on shifting reference frames, the event on particle A, viewed as simultaneous, abruptly changes to one which is later in time, according to A, by $(v/c^2)x_0$. Its designated space coordinate also changes abruptly. Again, all these effects are merely due to a change in the arbitrary method of designating events.

Case 2

In this case, we consider a theoretical point particle B, which moves with an x-component velocity of v relative to our initial reference frame S. We want to evaluate the changes to the time and space coordinates on B when we abruptly shift to another inertial reference frame S′ moving with velocity v (relative to S) which is the same as the x-component of B's velocity.

2(a)

We shall first consider the event on B regarded as simultaneous to S at the moment of the shift in reference frame. From applying the Lorentz transformation equations, we get: