where u_y and u_z are, respectively, the y and z velocity components of particle A according to S, and $\Delta t = (v/c^2)x_0$ is the change in the time coordinate according to S, between this event considered simultaneous to S′ and that — as in 1(a) — considered simultaneous to S.

An important value to note here is the change in the proper time on particle A (Δt_A) between these two events:

$$\Delta t_A = \Delta t = (v/c^2)x_0$$

This means that immediately on shifting reference frames, the event on particle A, viewed as simultaneous, abruptly changes to one which is later in time, according to A, by $(v/c^2)x_0$. Its designated space coordinate also changes abruptly. Again, all these effects are merely due to a change in the arbitrary method of designating events.

Case 2

In this case, we consider a theoretical point particle B, which moves with an x-component velocity of v relative to our initial reference frame S. We want to evaluate the changes to the time and space coordinates on B when we abruptly shift to another inertial reference frame S′ moving with velocity v (relative to S) which is the same as the x-component of B's velocity.

2(a)

We shall first consider the event on B regarded as simultaneous to S at the moment of the shift in reference frame. From applying the Lorentz transformation equations, we get:

$$t = t_0, \quad x = x_0,$$

$$t' = t_0 - \frac{(v/c^2)x_0}{(1 - v^2/c^2)^{1/2}}, \quad x' = \frac{x_0}{(1 - v^2/c^2)^{1/2}},$$

$$y = y' = y_0, \quad z = z' = z_0.$$

2(b)

Here, we consider the event on B regarded as simultaneous to S′ at the moment of the shift in reference frame. We obtain:

$$t = t_0 + \frac{(v/c^2)x_0}{1 - v^2/c^2}, \quad x = \frac{x_0}{1 - v^2/c^2},$$

$$t' = t_0, \quad x' = \frac{x_0}{(1 - v^2/c^2)^{1/2}},$$

$$y = y' = y_0 + u_y\Delta t, \quad z = z' = z_0 + u_z\Delta t$$

where $\Delta t = \dfrac{(v/c^2)x_0}{1 - v^2/c^2}$ is the change in the time coordinate according to S between this event and that considered in 2(a).

In this case, Δt_B which is the change in the proper time on particle B between the two events considered in 2(a) and 2(b) is not the same as Δt. We have:

$$\Delta t_B = \frac{(v/c^2)x_0}{(1 - v^2/c^2)^{1/2}}$$

With the knowledge of how the time and space coordinates of a particle alter during an abrupt shift of reference frame as in cases 1 and 2 given above, we can work out the changes for any inertial particle. A general procedure would be to first consider the changes in the time and space coordinates caused by a shift to a reference frame having the same velocity as the particle, and then add on the changes brought about by a subsequent shift to the actual final reference frame you want to consider. Other appropriate combinations may of course be employed.

For conceptual purposes, we shall now consider the case where we keep the time and space coordinates unchanged, when we shift from one inertial reference frame to another. We want to determine the change in the time and space coordinates according to the initial reference frame S, which would be required to correlate with the new event designated by the new reference frame S′. (S′ moves with velocity v relative to S along the x-axis.)

We therefore set:

$$t' = t_0, \qquad x' = x_0, \qquad y' = y_0, \qquad z' = z_0.$$

From the Lorentz transformation equations, we now obtain:

$$t = t_0 + \frac{(v/c^2)x_0}{(1 - v^2/c^2)^{1/2}},$$

$$x = \frac{x_0}{(1 - v^2/c^2)^{1/2}}, \qquad y = y_0, \qquad z = z_0.$$

With all the above results, we are now ready to finally "solve" the famous twin paradox.