V

The Twin Paradox

The twin paradox poses a problem in interpretation mainly because of man's assumption that time is a real entity that is pervasive. This notion then compels him to connect all the events experienced by each of the twins into simultaneous sets. He is then alarmed to find that, according to the "travelling" twin, there are events experienced by the "stationary" twin which cannot be reasonably linked by simultaneity, and may even seem to have disappeared (ie. there has been a sudden time leap). If we accept that time and space are not real pervasive entities but only arbitrary abstractions, the problem ceases to exist, as we can now demonstrate.

To simplify matters, we shall consider the case where no acceleration is involved and, instead of using twins, we shall compare the times shown on clocks. Consider Fig. 4. Rocket A approaches a stationary position E at constant velocity v. As

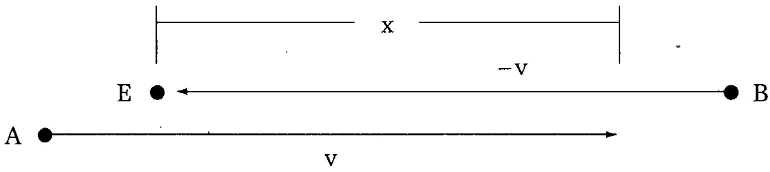

Fig. 4. Scenario for considering the twin paradox: Rocket A travels with velocity v past a stationary observer E to reach rocket B travelling in the opposite direction towards E with velocity −v. Rocket A passes rocket B at a distance x away from E, according to E. The equivalent distance according to either A or B is shorter by a factor of $(1 - v^2/c^2)^{1/2}$. Rocket A synchronizes his clock to that of E as he passes E, and rocket B synchronizes his clock to that of A as he passes A. When B arrives at E, their clock readings are compared.

it passes E, the clocks on A and E are synchronized. Rocket B, meanwhile, approaches E from the opposite direction at the velocity −v. As A and B pass by each other (at a distance x according to E) the clock on rocket B is synchronized to that on A. When B eventually reaches E, the clock readings are then compared. It will be found that a shorter time has elapsed on the rockets according to the clock on B when compared to the time shown on the clock on E.

Let us first consider the account of what has happened according to the observer on E. He will have considered both the clocks on A and B to have run slower by a factor of $(1 - v^2/c^2)^{1/2}$. Hence, according to E, we have:

$$\Delta t = 2x/v$$

$$\Delta t' = \Delta t(1 - v^2/c^2)^{1/2} = (2x/v)(1 - v^2/c^2)^{1/2}$$

where Δt is the time lapse according to E, between rocket A passing E and rocket B arriving at E. $\Delta t'$ is the corresponding time lapse recorded by the clocks on the rockets.

Now, we shall look at the situation according to the observers on rocket A and then on rocket B. We shall divide our analysis into three phases as follows:

Phase 1: between A passing E and A meeting B.
Phase 2: the shift of reference frame from A to B.
Phase 3: between B passing A and B meeting E.

We denote Δt_1, Δt_2, Δt_3 as the time lapses at E and $\Delta t_1'$, $\Delta t_2'$, $\Delta t_3'$ as the time lapses on the rockets corresponding to phases 1, 2 and 3 respectively.

In phase 1, the observer on A will have considered the clock on E to have run slower by a factor of $(1 - v^2/c^2)^{1/2}$. Due to the length contraction effect of the Lorentz transformation equations, he will also have considered the distance travelled in phase 1 to be $x(1 - v^2/c^2)^{1/2}$. Hence, for phase 1, we obtain:

$$\Delta t_1 = (x/v)(1 - v^2/c^2)$$

$$\Delta t_1' = (x/v)(1 - v^2/c^2)^{1/2}$$

In phase 2, we have to consider the effect on the time and space coordinates, brought about by the abrupt shift in reference frame from rocket A to rocket B. We shall evaluate the effects in two stages.

In the first stage, we determine the effects resulting from a shift from A to the reference frame stationary relative to E. We therefore apply the results from case 2(b) of section IV and obtain:

$$\Delta \tau_1 = vx/c^2$$

$$L_1 = x$$

where $\Delta\tau_1$ is the proper time lapse (according to E) between the event on E considered simultaneous to A and that considered simultaneous to the new reference frame. L_1 is the distance to E according to the new reference frame.

For the second stage, we have to evaluate the effect on the time and space coordinates resulting from an abrupt shift from the reference frame stationary relative to E to the reference frame of rocket B. We now apply the results from case 1(b) of section IV and obtain:

$$\Delta\tau_2 = vx/c^2$$

$$L_2 = x(1 - v^2/c^2)^{1/2}$$

where $\Delta\tau_2$ is the proper time lapse (according to E) between the event on E considered simultaneous to the reference frame stationary relative to E and that considered simultaneous to B. L_2 is the distance to E according to rocket B. We note that this is the same as that according to rocket A.

Hence, for phase 2, we obtain:

$$\Delta t_2 = \Delta\tau_1 + \Delta\tau_2 = 2vx/c^2$$

$$\Delta t_2' = 0$$

The fact that Δt_2 is not zero means that there has been an apparent instantaneous jump in time at E the moment the switch of reference frames occurred. It should also be noted that Δt_2 is directly proportional to the distance between E and the position where the rockets passed each other. This is one of the effects which many either had not considered or had found difficult to accept previously. However, once we understand that time and space are only arbitrary concepts and that all these changes are merely the result of an alteration in the method of designating events, such effects become perfectly acceptable.

In phase 3, the conditions are similar to those in phase 1 and we have:

$$\Delta t_3 = (x/v)(1 - v^2/c^2)$$

$$\Delta t_3' = (x/v)(1 - v^2/c^2)^{1/2}$$

Thus, according to the observers on A and B, the total time lapse at E is:

$$\Delta t_1 + \Delta t_2 + \Delta t_3 = 2x/v$$

and the total time lapse according to the clocks on the rockets is:

$$\Delta t_1' + \Delta t_2' + \Delta t_3' = (2x/v)(1 - v^2/c^2)^{1/2}$$

These results are exactly the same as those obtained from the point of view of the observer on E. Hence there is no paradox as both sets of observers agree on the results (ie. both agree that the time lapse on the rockets is less than that on E).